



7-4

Factoring Polynomials

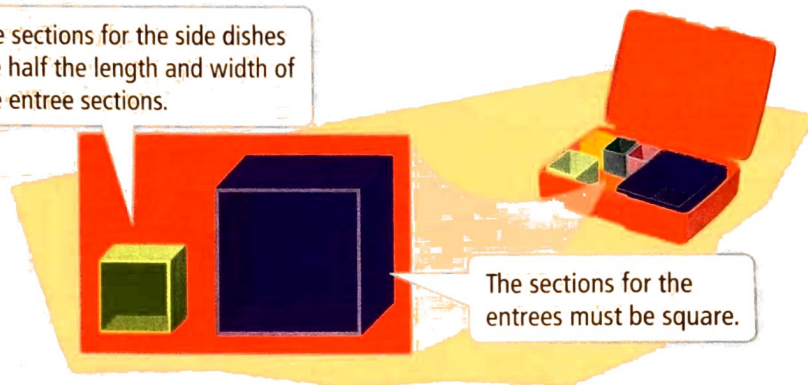
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I CAN... factor a polynomial.

MODEL & DISCUSS

A catering company has been asked to design meal boxes for entrees and side dishes.

The sections for the side dishes are half the length and width of the entree sections.



The sections for the entrees must be square.

- A. Design a meal box that meets each of these requirements:
- Equal numbers of sections for entrees and side dishes
 - More sections for entrees than for side dishes
 - More sections for side dishes than for entrees
- B. **Use Structure** For each meal box from Part A, write an algebraic expression to model the area of the meal boxes. © MP.7

ESSENTIAL QUESTION

How is factoring a polynomial similar to factoring integers?

EXAMPLE 1 Find the Greatest Common Factor

What is the greatest common factor (GCF) of the terms of $12x^5 + 8x^4 - 6x^3$?

Step 1 Write the prime factorization of the coefficient for each term to determine if there is a greatest common factor other than 1.

$$\begin{array}{ccc} 12 & 8 & 6 \\ \downarrow & \downarrow & \downarrow \\ 2 \cdot 2 \cdot 3 & 2 \cdot 2 \cdot 2 & 2 \cdot 3 \end{array}$$

One instance of 2 is the only common factor of the numbers, so the GCF of the coefficients of this trinomial is 2.

Step 2 Determine the greatest common factor for the variables of each term.

$$\begin{array}{ccc} x^5 & x^4 & x^3 \\ \downarrow & \downarrow & \downarrow \\ x \cdot x \cdot x \cdot x \cdot x & x \cdot x \cdot x \cdot x & x \cdot x \cdot x \end{array}$$

Three instances of x are the only common factors of the terms, so the GCF of the variables is x^3 .

The greatest common factor of $12x^5 + 8x^4 - 6x^3$ is $2x^3$.

Try It! 1. Find the GCF of the terms of each polynomial.

a. $15x^2 + 18$

b. $-18y^4 + 6y^3 + 24y^2$

CONCEPTUAL UNDERSTANDING

EXAMPLE 2 Factor Out the Greatest Common Factor

Why is it helpful to factor out the GCF from a polynomial?

Consider the polynomial $-12x^3 + 18x^2 - 27x$.

Step 1 Find the GCF of the terms of the polynomial, if there is one.

Because the first term is negative, it is helpful to factor out -1 .

COMMON ERROR

Remember to include the negative sign when factoring out the GCF of negative terms. Also, factoring out a -1 from a positive term generates two negative factors.

$$\begin{array}{ccc}
 -12x^3 & & 18x^2 & & -27x \\
 \swarrow & & \swarrow & & \swarrow \\
 -1 \cdot 2 \cdot 2 \cdot 3 \cdot x \cdot x \cdot x & & -1 \cdot (-2) \cdot 3 \cdot 3 \cdot x \cdot x & & -1 \cdot 3 \cdot 3 \cdot x
 \end{array}$$

The greatest common factor is $-3x$.

Step 2 Factor the GCF out of each term of the polynomial.

$$-3x(4x^2 - 6x + 9)$$

Factoring out the greatest common factor results in a polynomial with smaller coefficients and/or smaller exponents of the variable(s). This makes it easier to analyze the polynomial or factor it further.

Try It! 2. Factor out the GCF from each polynomial.

a. $x^3 + 5x^2 - 22x$

b. $-16y^6 + 28y^4 - 20y^3$

APPLICATION

EXAMPLE 3 Factor a Polynomial Model

Alani is in charge of marketing for a travel company. She is designing a brochure that will have 6 photos. The photos can be arranged on the page in a number of ways.

There are 2 main square photos, which have a length of x in. on each side.



There are 4 long pictures that are each 1 in. by x in.



A. What is the total area of the photos?

First, find the area of each type of photo.

Area = area of square photos + area of long photos

$$\begin{aligned}
 &= 2(x^2) + 4(1x) \\
 &= 2x^2 + 4x
 \end{aligned}$$

The total area of the photos is $2x^2 + 4x$.

There are 2 square photos, each with an area of x^2 in.². There are 4 thinner photos, each with an area of $1x$ in.².

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Activity



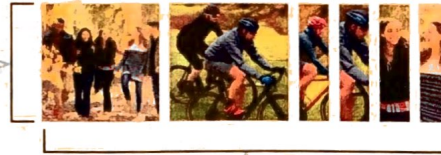
Assess

EXAMPLE 3 CONTINUED

- B. Find a rectangular arrangement for the photos. What factoring represents the arrangement?**

Try placing the photos in one row.

The arrangement has a height of x in.



The arrangement has a width of $(2x + 4)$ in.

The factoring that represents the arrangement is $x(2x + 4)$.

- C. Factor out the GCF from the polynomial. What does the GCF represent in this situation?**

The GCF of $2x^2$ and $4x$ is $2x$. So you can rewrite the expression as $2x(x + 2)$.

The arrangement has a height of $2x$ in.



The arrangement has a width of $(x + 2)$ in.

The GCF represents the height of one possible arrangement of the photos.

- D. Which of these two arrangements is a more practical use of the space on a page of the brochure?**

The arrangement based on the GCF is more practical because the arrangement with the photos in one line will likely be too wide for a page.



- Try It!** 3. Suppose the dimensions of the narrower photos were increased to 2 in. by x in. What expression would represent the new arrangement based on the GCF?

MODEL WITH MATHEMATICS

Think about how to represent this situation mathematically. How is the GCF useful in solving this problem? © MP.4

CONCEPT SUMMARY Factoring Polynomials

WORDS Determine if a polynomial can be factored. If the polynomial can be factored, find the greatest common factor of the terms and factor it out.

ALGEBRA

$$18x^3y^2 + 12x^2y + 15x$$

Find the GCF of the terms.

$$2 \cdot 3 \cdot 3 \cdot x \cdot x \cdot x \cdot y \cdot y \quad 2 \cdot 2 \cdot 3 \cdot x \cdot x \cdot y \quad 3 \cdot 5 \cdot x$$

The greatest common factor of $18x^3y^2 + 12x^2y + 15x$ is $3x$.

$$3x(6x^2y^2 + 4xy + 5)$$

Identify the remaining factors of the polynomial after factoring out the GCF, then write it in factored form.

Do You UNDERSTAND?

- ESSENTIAL QUESTION** How is factoring a polynomial similar to factoring integers?
- Look for Relationships** Why does the GCF of the variables of a polynomial have the *least* exponent of any variable term in the polynomial? © MP.7
- Vocabulary** What is the greatest common factor of two polynomials that do not appear to have any common factors?
- Error Analysis** Andrew factored $3x^2y - 6xy^2 + 3xy$ as $3xy(x - 2y)$. Describe and correct his error. © MP.3
- Error Analysis** Wendell says that the greatest common factor of x^6 and x^8 is x^2 , since the greatest common factor of 6 and 8 is 2. Is Wendell correct? Explain. © MP.3

Do You KNOW HOW?

Find the GCF of each pair of monomials.

- $10x$ and 25
- x^3y^2 and x^5y
- $8a^2$ and $28a^5$
- $4x^3$ and $9y^5$
- $12a^5b$ and $16a^4b^2$
- $14x^{10}y^8$ and $15x^6y^9$

Factor out the GCF from each polynomial.

- $10a^2b + 12ab^2$
- $-3x^4 + 12x^3 - 21x^2$
- $15x^3y - 10x^2y^3$
- $x^{10} + x^9 - x^8$
- $3x^3y^2 - 9xz^4 + 8y^2z$
- $100a^7b^5 - 150a^8b^3$



UNDERSTAND

18. **Use Structure** What term and $12x^2y$ have a GCF of $4xy^2$? Write an expression that shows the monomial factored out of the polynomial. © MP.7
19. **Look for Relationships** Write a trinomial that has a GCF of $4x^2$. © MP.7
20. **Error Analysis** Describe and correct the error a student made when factoring $10a^3b - 5a^2b^2 - 15ab$. © MP.3

$$10a^3b - 5a^2b^2 - 15ab$$

$$5a(2a^2b - ab^2 - 3b)$$



21. **Make Sense and Persevere** Write the difference in factored form.
 $(24x^4 - 15x^2 + 6x) - (10x^4 + 5x^2 - 4x)$ © MP.1
22. **Higher Order Thinking** In the expression $ax^2 + b$, the coefficients of a and b are multiples of 2.
The coefficients c and d in the expression $cx^2 + d$ are multiples of 3.
Will the GCF *always, sometimes, or never* be a multiple of 6? Explain.
23. **Make Sense and Persevere** What is the GCF in the expression $x(x + 5) - 3x(x + 5) + 4(x + 5)$? © MP.1
24. **Look for Relationships** Find the greatest common factor of the terms $x^{n+1}y^n$ and $x^n y^{n-2}$, where n is a whole number greater than 2. How can you factor the expression $x^{n+1}y^n + x^n y^{n-2}$? © MP.7

PRACTICE

Find the GCF of each group of monomials.

SEE EXAMPLE 1

25. $8y^3$ and $28y$
26. $9a^2b^3$, $15ab^2$, and $21a^4b^3$
27. $18m^2$ and 25
28. x^2y^3 and x^3y^5

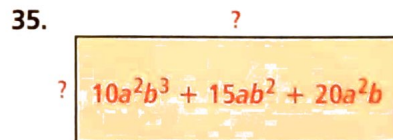
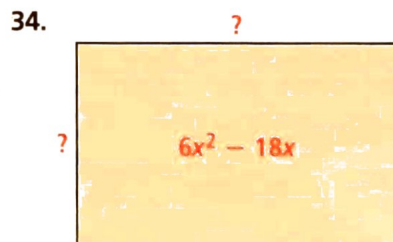
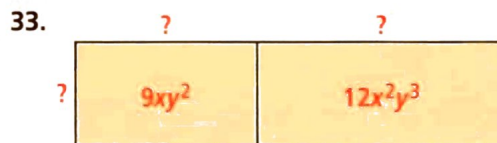
Factor out the GCF from each polynomial.

SEE EXAMPLE 2

29. $12x^2 - 15x$
30. $-4y^4 + 6y^2 - 14y$
31. $3m^2 - 10m + 4$
32. $24x^3y^2 - 30x^2y^3 + 12x^2y^4$

The areas of the rectangles are given. Use factoring to find expressions for the missing dimensions.

SEE EXAMPLE 3



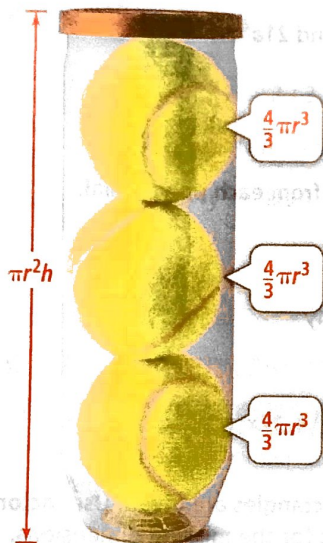
36. A farmer wants to plant three rectangular fields so that the widths are the same. The areas of the fields, in square yards, are given by the expressions $12x^2y$, $9xy^2$, and $21xy$. What is the width of the fields if $x = 3$ and $y = 4$?

SEE EXAMPLE 3

PRACTICE & PROBLEM SOLVING

APPLY

37. **Model With Mathematics** Write an expression in factored form to represent the volume in the canister not occupied by the tennis balls. © MP.4



38. **Use Structure** Determine the GCF and write the expression in factored form. © MP.7
 $(6x^2 + 4x) + (4x^2 - 8x)$
39. **Mathematical Connections** A sheet of dough has six identical circles cut from it. Write an expression in factored form to represent the amount of dough that is remaining. Is there enough dough for another circle?

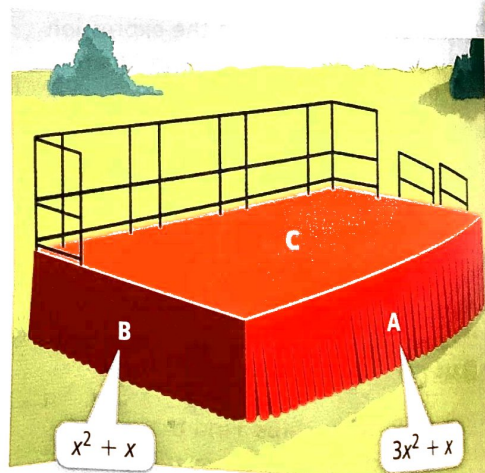


ASSESSMENT PRACTICE

40. Fill in the blanks to find the factor pairs for $18x^4 + 12x^3 - 24x^2$.

[]	$6x^2 + 4x - 8$
$2x$	$[]x^l + []x^l - []x$
x^l	$18x^2 + 12x - 24$
$[]x^l$	$3x^2 + 2x - 4$

41. **SAT/ACT** The area of a rectangle is $12x^3 - 18x^2 + 6x$. The width is equal to the GCF. What could the dimensions of the rectangle be?
 Ⓐ $6x(2x^2 - 3x)$
 Ⓑ $3(4x^3 - 6x^2 + 2x)$
 Ⓒ $x(12x^2 - 18x + 6)$
 Ⓓ $6x(2x^2 - 3x + 1)$
42. **Performance Task** Camilla is designing a platform for an athletic awards ceremony. The areas for two of the three faces of a platform are given.



- Part A What are the dimensions of each face of the platform?
- Part B What is the area of the top of the platform?
- Part C What expression represents the surface area of the platform?
- Part D What expression represents the volume of the platform?